Fall 2020	HW problems for Chapter 10	ENG 5300.00

You must show **all** work to receive full credit. No . All work is to be your own. Date: October 7
Be neat and organized, and use correct notation. All of these problems appear on Test 1. 18:40 - 19:55

1 §10.1 Line Integrals

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [z,\, x,\, y],\ \ C: \mathbf{r} = [\cos t,\, \sin t,\, t]$ from $(1, 0, 0)$ to $(1, 0, 4\pi)$. 10 points

2. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [e^x,\, e^y,\, e^z],\ \ C: \mathbf{r} = [t,\, t^2,\, t^2]$ from $(0, 0, 0)$ to $(2, 4, 4)$. 10 points

3. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x - y,\, y - z,\, z - x],\ \ C: \mathbf{r} = [2\cos t,\, t,\, 2\sin t]$ from $(2, 0, 0)$ to $(2, 2\pi, 0)$. 10 points

   Hint: $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

4. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x^2,\, y^2,\, z^2],\ \ C: \mathbf{r} = [\cos t,\, \sin t,\, e^t]$ from $(1, 0, 1)$ to $(1, 0, e^{2\pi})$. 10 points

5. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [e^{-x},\, e^{-y},\, e^{-z}],\ \ C: \mathbf{r} = [t,\, t^2,\, t]$ from $(0, 0, 0)$ to $(2, 4, 2)$. 10 points

6. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x + y,\, y + z,\, z + x],\ \ C: \mathbf{r} = [2t,\, 5t,\, t]$ from $t = -1$ to 1. 10 points

7. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x,\, -z,\, 2y]$, , from $(1, 2, 3)$ straight to $(3, 2, 1)$. 10 points

8. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x - y,\, y - z,\, z - x],\ \ C: \mathbf{r} = [2\,\cos t,\, t,\, 2\,\sin t]$ from $(2, 0, 0)$ to $(2, 2\pi, 0)$. 10 points

9. §10.1 Line Integral. Work done by a force. Evaluate the line integral, where C is the given curve. (Show the details.) 10 points

$$\int\limits_C (y + z)dx + (x + z)dy + (x + y)dz, \ \ C \text{ is the line segment from } (1, 0, 1) \text{ to } (0, 1, 2)$$

10. §10.1 Line Integral. Work done by a force. Calculate $\int\limits_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.) 10 points
   $\mathbf{F} = \sin x\ \mathbf{i} + \cos y\ \mathbf{j} + xz\ \mathbf{k},\ \ C: \mathbf{r}(t) = t^3\ \mathbf{i} - t^2\ \mathbf{j} + t\ \mathbf{k}$ from $(0, 0, 0)$ to $(1, -1, 1)$.

2 §10.2 Path Independence of Line Integrals

1. §10.2 Path-Independent Integrals. Show that the form under the integral sign is exact in the space and evaluate the integral. (Show the details of your work). 10 points

$$\int_{(2,3,0)}^{(0,1,2)} (z\,e^{xz}\,dx + dy\, + xe^{xz}\,dz)$$

2. §10.2 Check for Path Independence and, if independent, integrate from $(0,0,0)$ to (a,b,c). (Show the details of your work.) 10 points

$$xy\,z^2\,dx + \frac{1}{2}x^2z^2\,dy + x^2yz\,dz$$

3. §10.2 Check for Path Independence and, if independent, integrate from $(0,0,0)$ to (a,b,c). (Show the details of your work.) 10 points

$$e^y\,dx + (xe^y - e^z)\,dy - ye^z\,dz$$

4. §10.2 Path Independent Integrals. Show the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(0,0,0)}^{(1,1,0)} e^{x^2+y^2+z^2}(x\,dx + y\,dy + z\,dz)$$

5. §10.2 Show the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(5,3,\pi)}^{(3,\pi,3)} (\cos yz\,dx - xz\sin yz\,dy - xy\sin yz\,dz)$$

6. §10.2 Check for Path Independence and, if independent, integrate from $(0,0,0)$ to (a,b,c). (Show the details of your work.) 10 points

$$(\cos(x^2 + 2y^2 + z^2))(2x\,dx + 4y\,dy\, + 2z\,dz)$$

7. §10.2 Show that the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(0,0,\pi)}^{(2,\frac{1}{2},\frac{\pi}{2})} e^{xy}(y\,\sin z\,dx + x\,\sin z\,dy + \cos z\,dz)$$

8. §10.2 Show that the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(0,1,0)}^{(1,0,1)} (e^x\cosh y\,dx + (e^x\sinh y + e^z\cosh y)\,dy + e^z\sinh y\,dz)$$

9. §10.2 Show that the field $\boxed{\mathbf{F}(x,y,z) = yze^{xz}\ \mathbf{i} + e^{xz}\ \mathbf{j} + xye^{xz}\ \mathbf{k}}$ is conservative and evaluate the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ along $C:\ \mathbf{r}(t) = (t^2+1)\ \mathbf{i} + (t^2-1)\ \mathbf{j} + (t^2-2t)\ \mathbf{k},\ \ 0 \le t \le 2$. Show the details of your work. 10 points

10. §10.2 Show that the field $\boxed{\mathbf{F}(x,y,z) = \sin y\ \mathbf{i} + (x\cos y + \cos z)\ \mathbf{j} - y\sin z\ \mathbf{k}}$ is conservative and evaluate the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ along $C:\ \mathbf{r}(t) = \sin t\ \mathbf{i} + t\ \mathbf{j} + 2t\ \mathbf{k},\ \ 0 \le t \le \frac{\pi}{2}$. Show the details of your work. 10 points

3 §10.4 Green's Theorem in the Plane

1. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\int_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [x^2+y^2, x^2-y^2]$, $R: 1 \le y \le 2-x^2$. Sketch R. 20 points

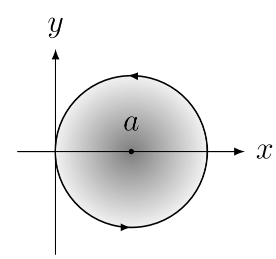
2. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\int\limits_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [2x-3y, x+5y]$, $R: 16x^2+25y^2 \le 400,\ \ y \ge 0$ 20 points
Hint: You might find the following identities useful:

$$\boxed{\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C} \text{ and } \boxed{\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C}$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint\limits_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [x^2e^y, y^2e^x]$, R the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$, $(0,3)$ 20 points

4. §10.4 Evaluation of Line Integrals by Green's Theorem.
Use Green's Theorem to evaluate 20 points

$$\oint\limits_C 3x^2y^2\,dx + 2x^2(1+xy)\,dy$$



where C is the circle shown.

5. §10.4 Evaluation of Line Integrals by Green's Theorem.
   (a) Verify the identity $\nabla\cdot\nabla\times\mathbf{F} = 0$ 4 points
   (b) Verify the identity $\nabla\times\nabla f = \mathbf{0}$ 4 points
   (c) Using Green's Theorem, evaluate $\oint\limits_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where $\mathbf{F} = \text{grad}\ (x^3\cos^2(xy))$, $R:\ 1 \le y \le 2-x^2$. 12 points

6. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint\limits_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [x^2y^2,\ -x/y^2]$, $R:\ 1 \le x^2+y^2 \le 4,\ x \ge 0,\ y \ge x$. 20 points

Hint: $\int \frac{1}{\sin^2\theta}d\theta = -\frac{\cos\theta}{\sin\theta} + C$

7. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [\cosh y, -\sinh x]$, $R:\ 1 \le x \le 3,\quad x \le y \le 3x$ 20 points

8. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [x\ \cosh 2y, 2x^2 \sinh 2y]$, $\quad R:\ x^2 \le y \le x.$ 20 points

9. §10.4 Evaluation of Line Integrals by Green's Theorem.
Using Green's Theorem, evaluate $\boxed{\int_C xy^2\, dx + 2x^2 y\, dy}$ counterclockwise around the boundary curve C. Where C is the triangle with vertices $(0,0), (2,2), (2,4)$. 20 points

10. §10.4 Evaluation of Line Integrals by Green's Theorem.
Using Green's Theorem, evaluate $\boxed{\int_C y^3\, dx - x^3\, dy}$ counterclockwise around the boundary curve C of the region R, where C is the circle $\boxed{x^2 + y^2 = 4}$. 20 points

4 §10.6 Surface Integrals

1. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$. Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [x\,,\, y\,,\, z]$, $S:\ \mathbf{r} = [u\ \cos v\,,\ u\ \sin v\,,\ u^2]$, $0 \le u \le 4$, $-\pi \le v \le \pi$

2. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$. Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [y^3\,,\, x^3\,,\, z^3]$, $S:\ x^2 + 4y^2 = 4,\ \ x \ge 0,\ \ y \ge 0,\ \ 0 \le z \le h$

3. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
$\mathbf{F} = [y^2\,,\, x^2\,,\, z^4]$, $S:\ z = 4\sqrt{x^2+y^2},\ \ 0 \le z \le 8,\ \ y \ge 0$

4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$ Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [0\,,\, \sin y\,,\, \cos z]$, S the cylinder $x = y^2$, where $0 \le y \le \frac{\pi}{4}$ and $0 \le z \le y$

5. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
$\mathbf{F} = [0,\ \sinh z,\ \cosh x]$, $S:\ x^2 + z^2 = 4,\ \ 0 \le x \le \sqrt{2},\ \ 0 \le y \le 5,\ \ z \ge 0$

6. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$ Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [\tan xy,\ x,\ y]$, $S:\ \ y^2 + z^2 = 1,\ \ 2 \le x \le 5,\ \ y \ge 0,\ \ z \ge 0$

7. §10.6 Flux Integrals (3) $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
$\mathbf{F} = [e^y,\ e^x,\ 1]$, $\quad S:\ x + y + z = 1,\ \ x \ge 0,\ \ y \ge 0,\ \ z \ge 0$

8. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [0,\, x,\, 0]$, $S:\ x^2 + y^2 + z^2 = 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$

9. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$. Evaluate $\boxed{\iint_S x^2\, dydz + y^2\, dxdz + z^2\, dxdy}$. 20 points
Where S is the round portion of $0 \leq z \leq \sqrt{1 - y^2}$, $0 \leq x \leq 2$. Describe the kind of surface. Show the details of your work.
Hint: $\boxed{\int \cos^3 t\ dt = \frac{1}{3}\cos^2 t \sin t + \frac{2}{3}\sin t + C}$ and $\boxed{\int \sin^3 t\ dt = -\frac{1}{3}\sin^2 t \cos t - \frac{2}{3}\cos t + C}$

10. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$. Evaluate $\boxed{\iint_S x\, dydz - z\, dxdz + y\, dxdy}$. 20 points
Where S a portion of $x^2 + y^2 + z^2 = 4$ in the first octant, oriented away from the origin. Describe the kind of surface. Show the details of your work.
Hint: $\boxed{\int \cos^3 t\ dt = \frac{1}{3}\cos^2 t \sin t + \frac{2}{3}\sin t + C}$ and $\boxed{\int \sin^3 t\ dt = -\frac{1}{3}\sin^2 t \cos t - \frac{2}{3}\cos t + C}$.
Moreover $\boxed{\cos^2 t = \frac{1}{2}(1 + \cos 2t)}$ and $\boxed{\sin^2 t = \frac{1}{2}(1 - \cos 2t)}$

5 §10.7 Triple Integrals. Divergence Theorem of Gauß

1. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

   Evaluate the integral by the Divergence Theorem. (Show the details.)
   $\mathbf{F} = [z - y\,,\, y^3\,,\, 2z^3]$, S the surface of $y^2 + z^2 \leq 4$, $-3 \leq x \leq 3$

2. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

   Evaluate the integral by the Divergence Theorem. (Show the details.)
   $\mathbf{F} = [5x^3\,,\, 5y^3\,,\, 5z^3]$, $S:\ x^2 + y^2 + z^2 = 4$
   Hint: The following facts might be useful:

   $$\begin{aligned} \text{Cartesian coordinates: } & dV = dx\, dy\, dz \\ \text{Cylindrical coordinates: } & dV = r\, dr\, d\theta\, dz,\ \ 0 \leq \theta \leq 2\pi,\ \ r \geq 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z \\ \text{Spherical coordinates: } & dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta,\ \ 0 \leq \theta \leq 2\pi,\ \ 0 \leq \phi \leq \pi,\ \ \rho \geq 0, \\ & x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi \end{aligned}$$

3. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

   Evaluate the surface integral by the Divergence Theorem. Show the details.
   $\mathbf{F} = [e^x\,,\, e^y\,,\, e^z]$, S the surface of the cube $|x| \leq 1$, $|y| \leq 1$, $|z| \leq 1$

4. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

   Evaluate the surface integral by the Divergence Theorem. Show the details.
   $\mathbf{F} = [\sin y\,,\, \cos x\,,\, \cos z]$, S, the surface of the cylinder and two disks: $x^2 + y^2 \leq 4$, $|z| \leq 2$

Hint: The following facts might be useful:

$$\begin{aligned}
\text{Cartesian coordinates: } & dV = dx\,dy\,dz \\
\text{Cylindrical coordinates: } & dV = r\,dr\,d\theta\,dz,\ \ 0 \le \theta \le 2\pi,\ \ r \ge 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z \\
\text{Spherical coordinates: } & dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta,\ \ 0 \le \theta \le 2\pi,\ \ 0 \le \phi \le \pi,\ \ \rho \ge 0, \\
& x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi
\end{aligned}$$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ — 20 points

   Evaluate the surface integral $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ by the Divergence Theorem. Show the details.

   $\mathbf{F} = [2x^2,\ \frac{1}{2}y^2,\ \sin\pi z]$, S the surface of the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$

6. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ — 20 points

   Evaluate the surface integral $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ by the Divergence Theorem. Show the details.

   $\mathbf{F} = [x^2,\ y^2,\ z^2]$, S, the surface of the cone: $x^2 + y^2 \le z^2$, $0 \le z \le h$

   Hint: The following facts might be useful:

$$\begin{aligned}
\text{Cartesian coordinates: } & dV = dx\,dy\,dz \\
\text{Cylindrical coordinates: } & dV = r\,dr\,d\theta\,dz,\ \ 0 \le \theta \le 2\pi,\ \ r \ge 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z \\
\text{Spherical coordinates: } & dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta,\ \ 0 \le \theta \le 2\pi,\ \ 0 \le \phi \le \pi,\ \ \rho \ge 0, \\
& x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi
\end{aligned}$$

7. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ — 20 points

   Evaluate the surface integral $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ by the Divergence Theorem. Show the details.

   $\mathbf{F} = [xy,\ yz,\ zx]$, S the surface of the cone $x^2 + y^2 \le 4z^2$, $0 \le z \le 2$

   Hint: The following facts might be useful:

$$\begin{aligned}
\text{Cartesian coordinates: } & dV = dx\,dy\,dz \\
\text{Cylindrical coordinates: } & dV = r\,dr\,d\theta\,dz,\ \ 0 \le \theta \le 2\pi,\ \ r \ge 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z \\
\text{Spherical coordinates: } & dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta,\ \ 0 \le \theta \le 2\pi,\ \ 0 \le \phi \le \pi,\ \ \rho \ge 0, \\
& x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi
\end{aligned}$$

8. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ — 20 points

   Evaluate the surface integral $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ by the Divergence Theorem. Show the details.

   $\mathbf{F} = [x^3 - y^3,\ y^3 - z^3,\ z^3 - x^3]$, S, the surface of $x^2 + y^2 + z^2 \le 25$, $z \ge 0$

   Hint: The following facts might be useful:

$$\begin{aligned}
\text{Cartesian coordinates: } & dV = dx\,dy\,dz \\
\text{Cylindrical coordinates: } & dV = r\,dr\,d\theta\,dz,\ \ 0 \le \theta \le 2\pi,\ \ r \ge 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z \\
\text{Spherical coordinates: } & dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta,\ \ 0 \le \theta \le 2\pi,\ \ 0 \le \phi \le \pi,\ \ \rho \ge 0, \\
& x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi
\end{aligned}$$

9. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

   Evaluate the surface integral $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the Divergence Theorem. Show the details.

   $\mathbf{F} = [\cos z + xy^2,\ xe^{-z},\ \sin y + x^2 z]$, S the surface of the solid bounded by $z = x^2 + y^2$ and the plane $z = 4$.

10. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

    Evaluate the surface integral $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the Divergence Theorem. Show the details.

    $\mathbf{F} = [3xy^2,\ xe^z,\ z^3]$, S is the surface of the solid bounded by $\boxed{y^2 + z^2 = 1}$ and $\boxed{x = -1}$, and $\boxed{x = 2}$

6 §10.9 Stokes's Theorem

1. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate the integral by Stokes's theorem for the following $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be Positive. (Show the details.)
   $\mathbf{F} = [y^2,\ x^2,\ -x + z]$, around the right triangle with vertices $(0, 0, 1)$, $(1, 0, 1)$, $(1, 1, 1)$

2. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [y^2,\ x^2,\ z + x]$, around the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$

3. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [-5y,\ 4x,\ z]$, C the circle $x^2 + y^2 = 16$, $z = 4$

4. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [z,\ e^z,\ 0]$, C the boundary curve of the portion of the cone $z = \sqrt{x^2 + y^2}$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 1$

5. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [0,\ z^3,\ 0]$, C the boundary curve of the cylinder $x^2 + y^2 = 1$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 1$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian

coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
$\mathbf{F} = [e^y,\ 0,\ e^x]$, C around the triangle with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$

7. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [z^3,\ x^3,\ y^3]$, C the circle $x = 2$, $y^2 + z^2 = 9$

8. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [yz,\ 2xz,\ e^{xy}]$, C the circle $x^2 + y^2 = 16$, $z = 5$

9. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

   Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.
   $\mathbf{F} = [x + y^2,\ y + z^2,\ z + x^2]$, C around the triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$