Fall 2020 ENG 5300 Test $1 \quad$ Mingzuoyang Chen

You must show all work to receive full credit. All work is to be your own.
This is a closed books and notes test. Be organized. Total points: 100 18:40-19:55
Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. $\S 10.1$ Line Integral. Work done by a force. Calculate $\int_{C} \mathbf{F}(\mathbf{r}) \cdot d \mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along $C$. (Show the details.) $\mathbf{F}=[x+y, y+z, z+x], C: \mathbf{r}=[2 t, 5 t, t]$ from $t=-1$ to 1 .

10 points
2. $\S 10.2$ Path-Independent Integrals. Show that the form under the integral sign is exact in the space and evaluate the integral. (Show the details of your work).

$$
\int_{(2,3,0)}^{(0,1,2)}\left(z e^{x z} d x+d y+x e^{x z} d z\right)
$$

3. $\S 10.4$ Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_{C} \mathbf{F}(\mathbf{r}) \cdot d \mathbf{r}$ counterclockwise around the boundary curve $C$ of the region $R$, where $\mathbf{F}=\left[x^{2} y^{2},-x / y^{2}\right], R: 1 \leq x^{2}+y^{2} \leq 4, x \geq 0, y \geq x$. 20 points Hint: $\int \frac{1}{\sin ^{2} \theta} d \theta=-\frac{\cos \theta}{\sin \theta}+C$
4. §10.6 Flux Integrals (3) $\iint_{S} \mathbf{F} \cdot \mathbf{n} d A$. Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.)

$$
\mathbf{F}=[x, y, z], S: \mathbf{r}=\left[u \cos v, u \sin v, u^{2}\right], 0 \leq u \leq 4,-\pi \leq v \leq \pi
$$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiiint_{S} \mathbf{F} \cdot \mathbf{n} d A$ 20 points

Evaluate the surface integral $\int \mathbf{F} \cdot \mathbf{n} d A$ by the Divergence Theorem. Show the details. $\mathbf{F}=\left[x^{2}, y^{2}, z^{2}\right], S$, the surface of the cone: $x^{2}+y^{2} \leq z^{2}, 0 \leq z \leq h$
6. $\S 10.9$ Evaluation of $\oint_{C} \mathbf{F} \cdot \mathbf{r}^{\prime} d s$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and $C$. Assume the Cartesian coordinates to be right-handed and the $z$-component of the surface normal to be nonnegative. Show the details.
$\mathbf{F}=\left[0, z^{3}, 0\right], C$ the boundary curve of the cylinder $x^{2}+y^{2}=1, x \geq 0, y \geq 0,0 \leq z \leq 1$

