Fall 2020 ENG 5300 Test 1 Jon-Michael X Grabowski

You must show **all** work to receive full credit. All work is to be your own. Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100** 18:40- 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r})\cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
$\mathbf{F} = [x-y,\ y-z,\ z-x]$, $C : \mathbf{r} = [2\ \cos t,\ t,\ 2\ \sin t]$ from $(2, 0, 0)$ to $(2, 2\pi, 0)$. 10 points

2. §10.2 Check for Path Independence and, if independent, integrate from $(0, 0, 0)$ to (a, b, c). (Show the details of your work.) 10 points

$$e^y\,dx + (xe^y - e^z)\,dy - ye^z\,dz$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [x \cosh 2y, 2x^2 \sinh 2y], \quad R: \ x^2 \leq y \leq x.$ 20 points

4. §10.6 Flux Integrals (3) $\iint\limits_S \mathbf{F}\cdot\mathbf{n}\,dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
$\mathbf{F} = [y^2\,,\, x^2\,,\, z^4]$, $S:\ z = 4\sqrt{x^2+y^2},\ \ 0 \le z \le 8,\ \ y \ge 0$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

Evaluate the surface integral $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the Divergence Theorem. Show the details.

$\mathbf{F} = [x^3 - y^3,\, y^3 - z^3,\, z^3 - x^3]$, S, the surface of $x^2 + y^2 + z^2 \leq 25,\ z \geq 0$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' \, ds$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [y^2, x^2, z + x]$, around the triangle with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$