Fall 2020 ENG 5300 Test 1 Chaowei Li

You must show **all** work to receive full credit. All work is to be your own. Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100** 18:40- 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r})\cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [z,\, x,\, y]$, $C : \mathbf{r} = [\cos t,\, \sin t,\, t]$ from $(1, 0, 0)$ to $(1, 0, 4\pi)$. 10 points

2. §10.2 Check for Path Independence and, if independent, integrate from $(0,0,0)$ to (a,b,c). (Show the details of your work.) 10 points

$$(\cos(x^2 + 2y^2 + z^2))(2x\,dx + 4y\,dy\ + 2z\,dz)$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\int_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where $\mathbf{F} = [x^2 + y^2, x^2 - y^2]$, $R: 1 \leq y \leq 2 - x^2$. Sketch R. 20 points

4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [\tan xy,\ x,\ y]$, $S:\ y^2 + z^2 = 1,\ 2 \le x \le 5,\ y \ge 0,\ z \ge 0$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ 20 points

Evaluate the integral by the Divergence Theorem. (Show the details.)
$\mathbf{F} = [z - y\,,\, y^3\,,\, 2z^3]$, S the surface of $y^2 + z^2 \le 4$, $-3 \le x \le 3$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' \, ds$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [0,\ z^3,\ 0]$, C the boundary curve of the cylinder $x^2 + y^2 = 1$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 1$