

You must show **all** work to receive full credit. All work is to be your own.

Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100**

18:40- 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ for the following data. If \mathbf{F} is a force, this gives the work done in the displacement along C . (Show the details.)

$\mathbf{F} = [e^x, e^y, e^z]$, $C : \mathbf{r} = [t, t^2, t^2]$ from $(0, 0, 0)$ to $(2, 4, 4)$.

10 points

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2. §10.2 Show that the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(0,0,\pi)}^{(2,\frac{1}{2},\frac{\pi}{2})} e^{xy}(y \sin z \, dx + x \sin z \, dy + \cos z \, dz)$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R , where

$$\mathbf{F} = [2x - 3y, x + 5y], R : 16x^2 + 25y^2 \leq 400, y \geq 0$$

20 points

Hint: You might find the following identities useful:

$$\boxed{\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C} \quad \text{and} \quad \boxed{\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C}$$

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4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n} dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
- $\mathbf{F} = [e^y, e^x, 1]$, $S : x + y + z = 1, x \geq 0, y \geq 0, z \geq 0$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$

20 points

Evaluate the integral by the Divergence Theorem. (Show the details.)

$$\mathbf{F} = [5x^3, 5y^3, 5z^3], \quad S : x^2 + y^2 + z^2 = 4$$

Hint: The following facts might be useful:

Cartesian coordinates: $dV = dx \, dy \, dz$

Cylindrical coordinates: $dV = r \, dr \, d\theta \, dz$, $0 \leq \theta \leq 2\pi$, $r \geq 0$, $x = r \cos \theta$, $y = r \sin \theta$, $z = z$

Spherical coordinates: $dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$, $0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq \pi$, $\rho \geq 0$,

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi$$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' ds$

20 points

Calculate this line integral by Stokes's theorem for the given \mathbf{F} and C . Assume the Cartesian coordinates to be right-handed and the z -component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [e^y, 0, e^x]$, C around the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$