Fall 2020 ENG 5300 Test 1 Brendan McClanahan

You must show **all** work to receive full credit. All work is to be your own. Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100** 18:40 - 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

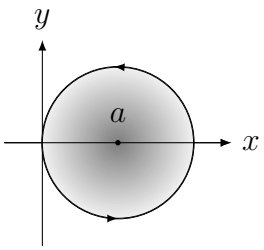
1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x^2,\ y^2,\ z^2],\ \ C : \mathbf{r} = [\cos t,\ \sin t,\ e^t]$ from $(1, 0, 1)$ to $(1, 0, e^{2\pi})$. 10 points

2. §10.2 Show that the field $\boxed{\mathbf{F}(x,y,z) = yze^{xz}\ \mathbf{i} + e^{xz}\ \mathbf{j} + xye^{xz}\ \mathbf{k}}$ is conservative and evaluate the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ along $C:\ \mathbf{r}(t) = (t^2+1)\ \mathbf{i}+(t^2-1)\ \mathbf{j}+(t^2-2t)\ \mathbf{k},\ \ 0 \le t \le 2$. Show the details of your work. 10 points

3. §10.4 Evaluation of Line Integrals by Green's Theorem.
Use Green's Theorem to evaluate 20 points

$$\oint_C 3x^2y^2\,dx + 2x^2(1+xy)\,dy$$

where C is the circle shown.



4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$. Evaluate $\boxed{\iint_S x^2\, dydz + y^2\, dxdz + z^2\, dxdy}$. 20 points

Where S is the round portion of $0 \leq z \leq \sqrt{1-y^2},\ 0 \leq x \leq 2$. Describe the kind of surface. Show the details of your work.

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

Evaluate the surface integral by the Divergence Theorem. Show the details.
$\mathbf{F} = [\sin y\,,\, \cos x\,,\, \cos z]$, S, the surface of the cylinder and two disks: $x^2 + y^2 \leq 4$, $|z| \leq 2$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [yz,\ 2xz,\ e^{xy}]$, C the circle $x^2 + y^2 = 16$, $z = 5$