

You must show **all** work to receive full credit. All work is to be your own.

Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100**

18:40 - 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ for the following data. If \mathbf{F} is a force, this gives the work done in the displacement along C . (Show the details.)
- $\mathbf{F} = [e^{-x}, e^{-y}, e^{-z}]$, $C : \mathbf{r} = [t, t^2, t]$ from $(0, 0, 0)$ to $(2, 4, 2)$. 10 points

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2. §10.2 Show that the field $\mathbf{F}(x, y, z) = \sin y \mathbf{i} + (x \cos y + \cos z) \mathbf{j} - y \sin z \mathbf{k}$ is conservative and evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ along $C : \mathbf{r}(t) = \sin t \mathbf{i} + t \mathbf{j} + 2t \mathbf{k}, 0 \leq t \leq \frac{\pi}{2}$. Show the details of your work. 10 points

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3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R , where $\mathbf{F} = \text{grad}(x^3 \cos^2(xy))$, $R: 1 \leq y \leq 2 - x^2$ 20 points

4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$. Evaluate $\iint_S x \, dydz - z \, dx dz + y \, dx dy$. 20 points

Where S a portion of $x^2 + y^2 + z^2 = 4$ in the first octant, oriented away from the origin. Describe the kind of surface. Show the details of your work.

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$

20 points

Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$ by the Divergence Theorem. Show the details.

$\mathbf{F} = [2x^2, \frac{1}{2}y^2, \sin \pi z]$, S the surface of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' ds$

20 points

Calculate this line integral by Stokes's theorem for the given \mathbf{F} and C . Assume the Cartesian coordinates to be right-handed and the z -component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [x + y^2, y + z^2, z + x^2]$, C around the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$