Fall 2020	ENG 5300	Test 1	Zachary Satawa

You must show **all** work to receive full credit. All work is to be your own. Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100** 18:40- 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r})\cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
$\mathbf{F} = [e^x,\, e^y,\, e^z]$, $C : \mathbf{r} = [t,\, t^2,\, t^2]$ from $(0,0,0)$ to $(2,4,4)$. 10 points

2. §10.2 Show that the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(0,0,\pi)}^{(2,\frac{1}{2},\frac{\pi}{2})} e^{xy}(y\ \sin z\, dx + x\ \sin z\, dy + \cos z\, dz)$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [2x - 3y, x + 5y]$, $R: 16x^2 + 25y^2 \leq 400,\ y \geq 0$ 20 points
Hint: You might find the following identities useful:

$$\boxed{\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C} \text{ and } \boxed{\int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C}$$

4. §10.6 Flux Integrals (3) $\iint\limits_S \mathbf{F}\cdot\mathbf{n}\,dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
$\mathbf{F} = [e^y,\ e^x,\ 1],\quad S:\ x+y+z=1,\ \ x \geq 0,\ \ y \geq 0,\ \ z \geq 0$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F}\cdot\mathbf{n}\,dA$ 20 points

Evaluate the integral by the Divergence Theorem. (Show the details.)
$\mathbf{F} = [5x^3\,,\,5y^3\,,\,5z^3]$, $S:\ x^2+y^2+z^2=4$
Hint: The following facts might be useful:

Cartesian coordinates: $dV = dx\,dy\,dz$

Cylindrical coordinates: $dV = r\,dr\,d\theta\,dz,\ \ 0\le\theta\le 2\pi,\ \ r\ge 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z$

Spherical coordinates: $dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta,\ \ 0\le\theta\le 2\pi,\ \ 0\le\phi\le\pi,\ \ \rho\ge 0,$
$x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' \, ds$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [e^y, \, 0, \, e^x]$, C around the triangle with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$