Fall 2020 ENG 5300 Test 1

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You must show **all** work to receive full credit. All work is to be your own.

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This is a closed books and notes test. Be organized. Total points: **100**

18:40 - 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x+y,\ y+z,\ z+x]$, $C : \mathbf{r} = [2t,\ 5t,\ t]$ from $t = -1$ to 1. 10 points

2. §10.2 Path-Independent Integrals. Show that the form under the integral sign is exact in the space and evaluate the integral. (Show the details of your work). 10 points

$$\int_{(2,3,0)}^{(0,1,2)} (z\, e^{xz}\, dx + dy + xe^{xz}\, dz)$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [x^2y^2,\ -x/y^2]$, $R:\ 1 \le x^2 + y^2 \le 4,\ x \ge 0,\ y \ge x$. 20 points
Hint: $\int \frac{1}{\sin^2\theta} d\theta = -\frac{\cos\theta}{\sin\theta} + C$

4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$. Evaluate the integral given below for the following data. Indicate the kind of surface. (Show the details of your work.) 20 points
$\mathbf{F} = [x\,,\, y\,,\, z]$, $S:\ \mathbf{r} = [u\,\cos v\,,\, u\,\sin v\,,\, u^2]$, $0 \le u \le 4$, $-\pi \le v \le \pi$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the Divergence Theorem. Show the details.

$\mathbf{F} = [x^2,\, y^2,\, z^2]$, S, the surface of the cone: $x^2 + y^2 \leq z^2$, $0 \leq z \leq h$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}'\, ds$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [0,\ z^3,\ 0]$, C the boundary curve of the cylinder $x^2 + y^2 = 1$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 1$