You must show **all** work to receive full credit. All work is to be your own. Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100** 18:40- 19:55

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate $\int_C \mathbf{F}(\mathbf{r})\cdot d\,\mathbf{r}$ for the following data. If $\mathbf{F}$ is a force, this gives the work done in the displacement along C. (Show the details.)
   $\mathbf{F} = [x,\ -z,\ 2y]$, , from $(1, 2, 3)$ straight to $(3, 2, 1)$. 10 points

2. §10.2 Check for Path Independence and, if independent, integrate from $(0,0,0)$ to (a,b,c). (Show the details of your work.) 10 points

$$xy\,z^2\,dx + \frac{1}{2}x^2z^2\,dy + x^2yz\,dz$$

3. §10.4 Evaluation of Line Integrals by Green's Theorem. Using Green's Theorem, evaluate $\oint_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where
$\mathbf{F} = [\cosh y, -\sinh x]$, $R:\ 1 \le x \le 3,\quad x \le y \le 3x$ 20 points

4. §10.6 Flux Integrals (3) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points

$\mathbf{F} = [e^y,\ e^x,\ 1],\quad S: x + y + z = 1,\ x \geq 0,\ y \geq 0,\ z \geq 0$

5. §10.7 Application of the Divergence Theorem: Surface Integrals $\oiint_S \mathbf{F} \cdot \mathbf{n}\, dA$ 20 points

Evaluate the integral by the Divergence Theorem. (Show the details.)
$\mathbf{F} = [5x^3 \,,\, 5y^3 \,,\, 5z^3]$, $S:\ x^2 + y^2 + z^2 = 4$
Hint: The following facts might be useful:

Cartesian coordinates: $dV = dx\, dy\, dz$

Cylindrical coordinates: $dV = r\, dr\, d\theta\, dz,\ \ 0 \le \theta \le 2\pi,\ \ r \ge 0,\ \ x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z$

Spherical coordinates: $dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta,\ \ 0 \le \theta \le 2\pi,\ \ 0 \le \phi \le \pi,\ \ \rho \ge 0,$
$x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi$

6. §10.9 Evaluation of $\oint_C \mathbf{F} \cdot \mathbf{r}' \, ds$ 20 points

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [e^y,\, 0,\, e^x]$, C around the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$