

You must show **all** work to receive full credit. All work is to be your own.

Oct 19 2020

This is a closed books and notes test. Be organized. Total points: **100**

**18:40- 19:55**

Submit to BB a single b/w pdf file, named using your last name. emailed solutions won't be graded

1. §10.1 Line Integral. Work done by a force. Calculate  $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$  for the following data. If  $\mathbf{F}$  is a force, this gives the work done in the displacement along  $C$ . (Show the details.) 10 points
- $\mathbf{F} = \sin x \mathbf{i} + \cos y \mathbf{j} + xz \mathbf{k}$ ,  $C : \mathbf{r}(t) = t^3 \mathbf{i} - t^2 \mathbf{j} + t \mathbf{k}$  from  $(0, 0, 0)$  to  $(1, -1, 1)$ .

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2. §10.2 Show the form under the integral sign is exact in space and evaluate the integral. Show the details of your work. 10 points

$$\int_{(5,3,\pi)}^{(3,\pi,3)} (\cos yz \, dx - xz \sin yz \, dy - xy \sin yz \, dz)$$

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3. §10.4 Evaluation of Line Integrals by Green's Theorem.

Using Green's Theorem, evaluate  $\int_C y^3 dx - x^3 dy$  counterclockwise around the boundary curve  $C$  of the region  $R$ , where  $C$  is the circle  $x^2 + y^2 = 4$ . 20 points

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4. §10.6 Flux Integrals (3)  $\iint_S \mathbf{F} \cdot \mathbf{n} dA$  Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work. 20 points
- $\mathbf{F} = [0, \sinh z, \cosh x]$ ,  $S : x^2 + z^2 = 4$ ,  $0 \leq x \leq \sqrt{2}$ ,  $0 \leq y \leq 5$ ,  $z \geq 0$

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5. §10.7 Application of the Divergence Theorem: Surface Integrals  $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$

20 points

Evaluate the surface integral  $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$  by the Divergence Theorem. Show the details.

$\mathbf{F} = [3xy^2, xe^z, z^3]$ ,  $S$  is the surface of the solid bounded by  $y^2 + z^2 = 1$  and  $x = -1$ , and  $x = 2$

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6. §10.9 Evaluation of  $\oint_C \mathbf{F} \cdot \mathbf{r}' ds$

20 points

Calculate this line integral by Stokes's theorem for the given  $\mathbf{F}$  and  $C$ . Assume the Cartesian coordinates to be right-handed and the  $z$ -component of the surface normal to be nonnegative. Show the details.

$\mathbf{F} = [z, e^z, 0]$ ,  $C$  the boundary curve of the portion of the cone  $z = \sqrt{x^2 + y^2}$ ,  $x \geq 0$ ,  $y \geq 0$ ,  $0 \leq z \leq 1$