You must show **all** work to receive full credit. All work is to be your own. Due: 11/30/2020

This is an open books and notes test. Be **very** organized. Total points: **100** 18:40- 19:55

Upload single *.zip folder with 1) written work in a single b/w pdf emailed solutions won't be graded
2) attach MATLAB scripts as *.m

1. Spectra of progressively narrowing impulses.

   (a) Find the Fourier series of 5 points

$$f_\alpha(x)=\begin{cases}0 & \text{if } \;-\pi\le x\le -\frac{\pi}{\alpha}\\ \frac{\alpha^2}{2\pi}x\sin(\alpha x) & \text{if } \;|x|\le\frac{\pi}{\alpha}\\ 0 & \text{if } \;+\frac{\pi}{\alpha}\le x\le +\pi\end{cases}\quad \text{and}\quad f_\alpha(x+2\pi)=f_\alpha(x)$$

   for $\alpha\in\{3.1, 9.1, 27.1\}$.

   (b) Use MATLAB to plot the Fourier coefficients and the graphs of $f_\alpha(x)$ for three values of $\alpha\in\{3.1, 9.1, 27.1\}$. 12 points

   (c) Show that area under the curve of $f_\alpha(x)$ over one period is independent of α. 4 points

   (d) From results above, what can be said about Fourier coefficients of $\lim\limits_{\alpha\to\infty} f_\alpha(x)$. 4 points

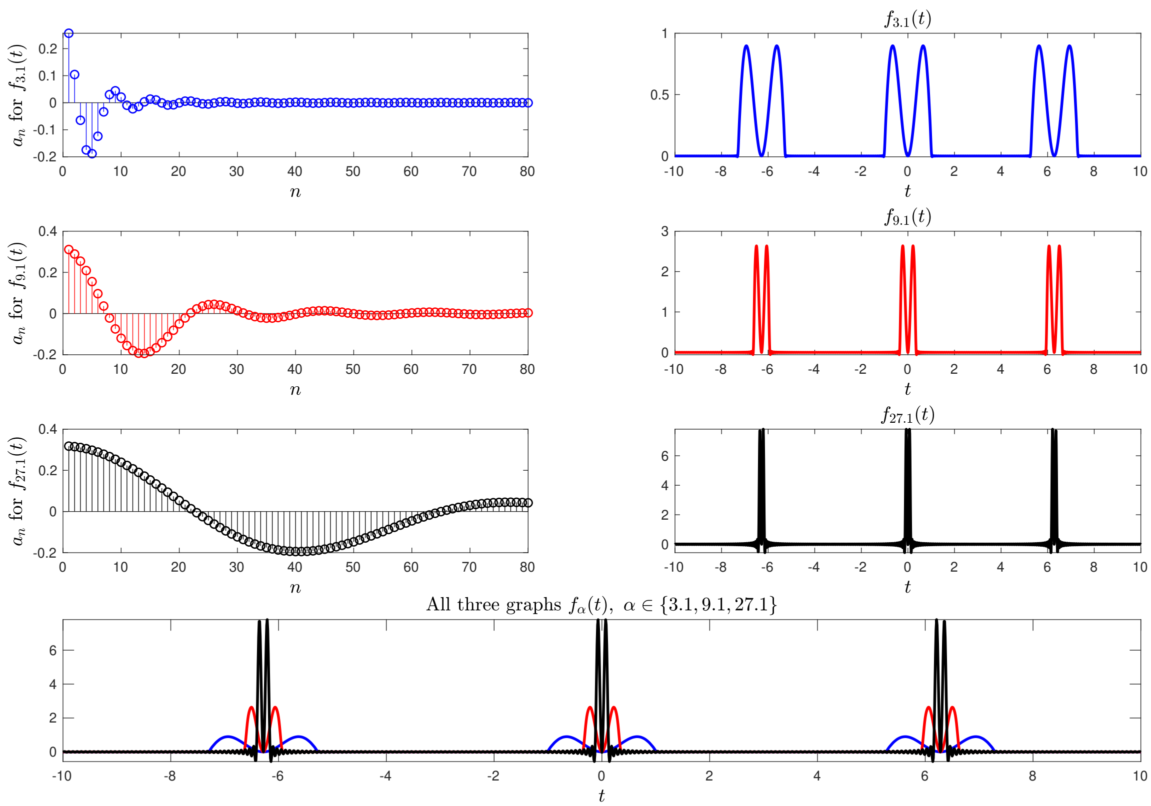


Figure 1: Your result in part (b) should look like this.

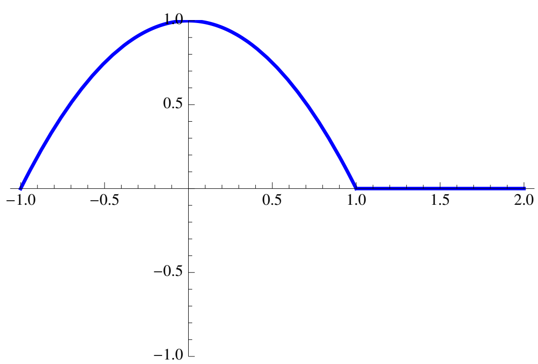
2. Effect of enlarging the period $p = 2L$. This is about discussion on pages 511–512. 20 points
For each of the following two functions, use MATLAB to plot the sum of the first 256 terms of the Fourier Series over the interval $-32 \le x \le 32$ for each $L \in \{3, 8, 16, 32\}$, and plot the spectrum a_n or b_n vs. $\omega_n = \frac{n\pi}{L}$ for $0 < \omega_n < 5\pi$.

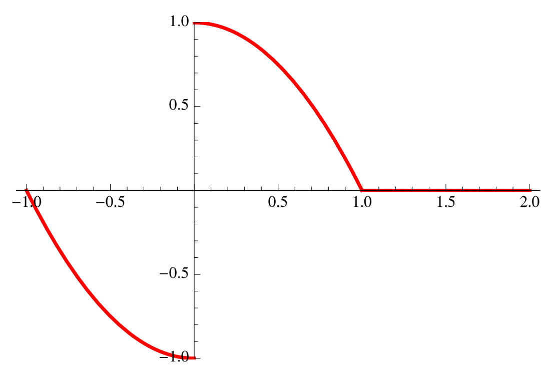
(a)

$$v_L(x) = \begin{cases} 0 & \text{if} \quad -L < x < -1 \\ 1 - x^2 & \text{if} \quad -1 < x < 1 \\ 0 & \text{if} \quad 1 < x < L \end{cases} \quad \text{and} \quad v_L(x) = v_L(x + 2L)$$

(b)

$$w_L(x) = \begin{cases} 0 & \text{if} \quad -L < x < -1 \\ x^2 - 1 & \text{if} \quad -1 < x < 0 \\ 1 - x^2 & \text{if} \quad 0 < x < 1 \\ 0 & \text{if} \quad 1 < x < L \end{cases} \quad \text{and} \quad w_L(x) = w_L(x + 2L)$$





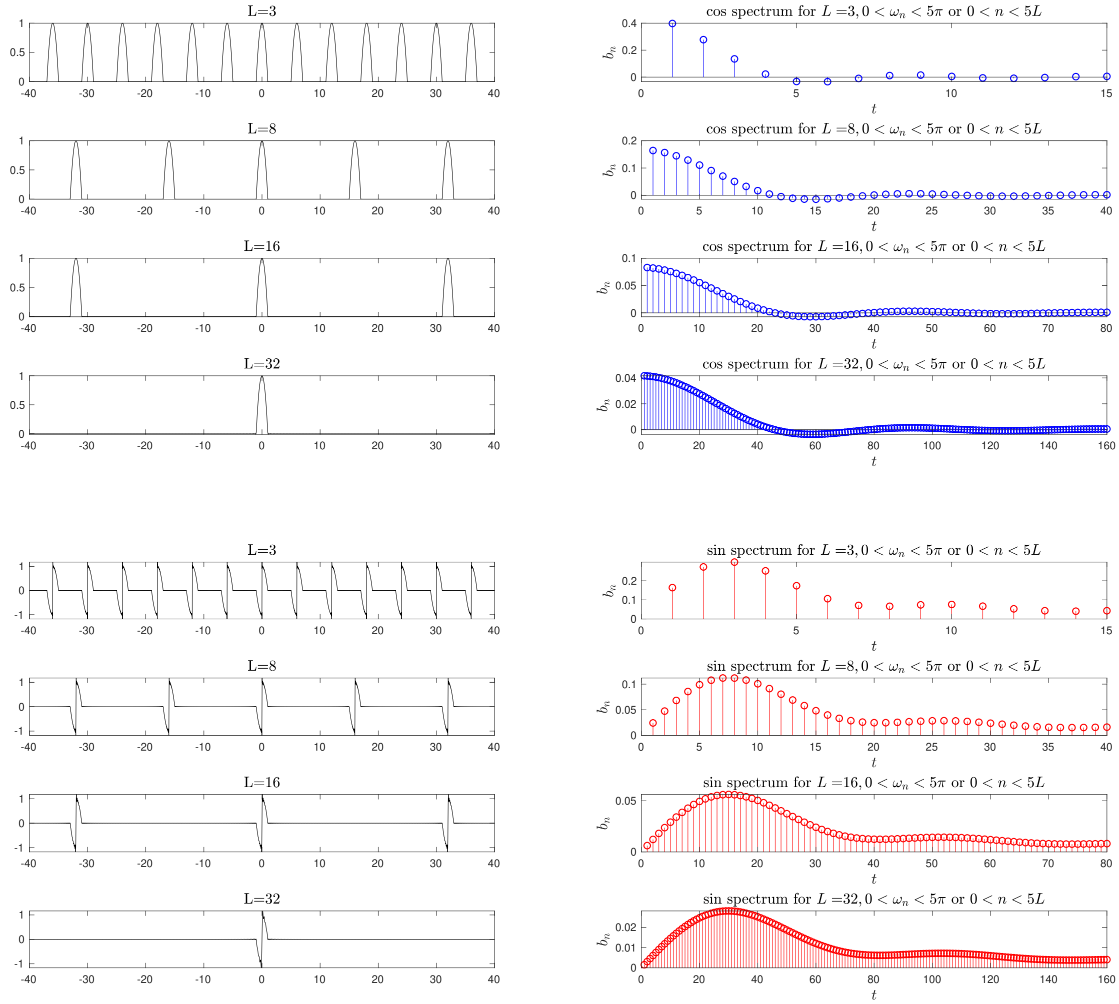


Figure 2: Your result in problem 2 should look like these.

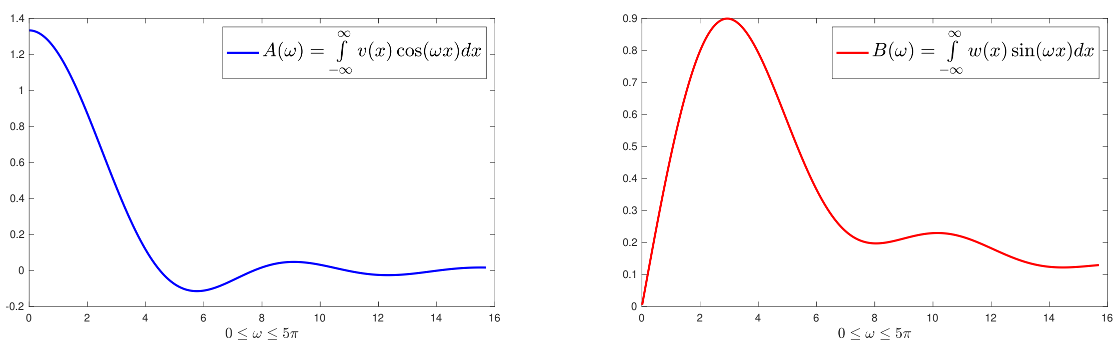


Figure 3: Your result in problem 3 should look like these.

3. For the functions in problem 2, set $L = \infty$, and

   (a) Find the Fourier cosine and sine transform(s), that is $A(\omega)$ for the even function, and $B(\omega)$ for the odd function. 6 points

   (b) Use MATLAB to plot $A(\omega)$ and $B(\omega)$ for for $0 < \omega < 5\pi$, and compare to the graphs of the Fourier coefficients in problem 2. 8 points

---

4. § 11.3 Forced Oscillations

   (a) Find the steady state oscillations of $\boxed{0.01y'' + 16y = f(t)}$ where 4 points

$$f(t) = \begin{cases} 0 & \text{if } -\frac{3}{2} \le t \le -1 \\ -\cos(\frac{\pi t}{2}) & \text{if } -1 \le t < \ \ 0 \\ +\cos(\frac{\pi t}{2}) & \text{if } \ \ 0 \le t \le +1 \\ 0 & \text{if } \ \ 1 \le t \le +\frac{3}{2} \end{cases} \quad \text{and} \quad f(t+3) = f(t)$$

   (b) Find the steady state solution to $\boxed{0.01y'' + \gamma y' + 16y = f(t)}$ with γ chosen to result in critically damped system. Call this solution $y_{cd}(t)$.

   (c) Plot[1] three curves on the same plane with $-3L \le t \le 3L$, one for $f(t)$ (in red), one for $5y(t)$ (in blue), and one for $5y_{cd}(t)$ (in green). 21 points

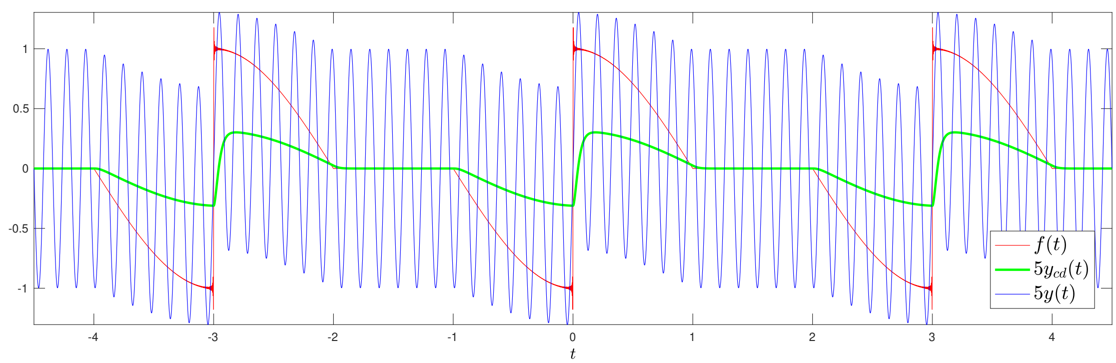


Figure 4: Your result should look like this.

[1]Comments on the plot:

i. Use MATLAB's legend to label the curves as shown in the figure.

ii. Use $n = 777$ terms in the Fourier Series.

iii. Plot $5y(t)$ instead of $y(t)$, and $5y_{cd}(t)$ instead of $y_{cd}(t)$ for scaling and visualization purposes.

5. §11.9 Properties of the Fourier Transform

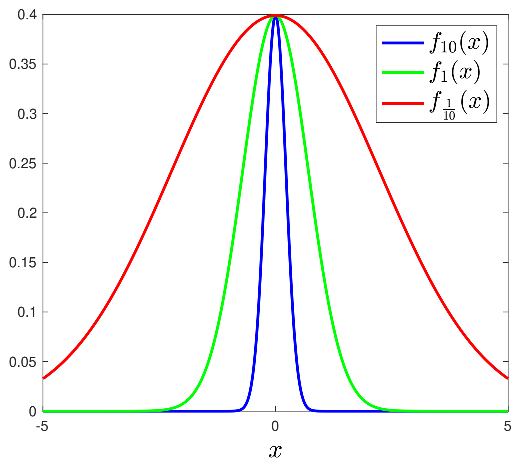
Let $\alpha > 0$ and consider

$$f_\alpha(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\alpha x^2}$$

(a) Find $\hat{f}_\alpha(\omega)$, the Fourier transform of $f_\alpha(x)$. 2 points

(b) Show that area under the curve of $\hat{f}_\alpha(\omega)$ is independent of α. 2 points

(c) Use MATLAB to sketch the graphs of $f_\alpha(x)$ and $\hat{f}_\alpha(\omega)$ for $\alpha \in \left\{10, 1, \frac{1}{10}\right\}$. 8 points



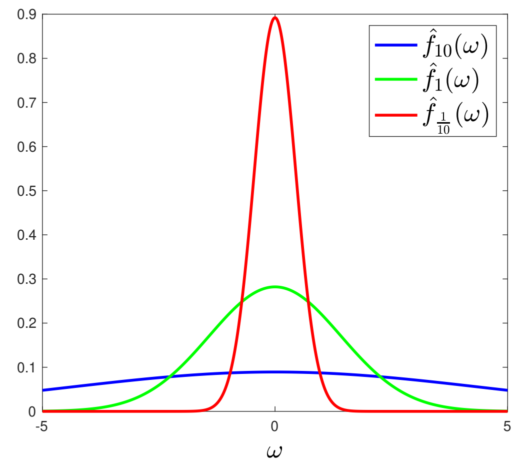


Figure 5: Your result should look like this.

(d) Describe both $\boxed{\lim_{\alpha \to 0} f_\alpha(x)}$ and $\boxed{\lim_{\alpha \to 0} \hat{f}_\alpha(\omega)}$ 4 points