Win 2022	Basics of Kalman Filter	ELEE 5810

You must show **all** work to receive full credit. All work is to be your own. Be neat and organized, and use correct notation.

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The purpose of this computer assignment is to get familiar with the basic Kalman Filter equations and their use.
Recall that the linear dynamical system (in discrete time) is specified by state update and measurement equations.

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t \quad \text{with} \quad \mathbb{E}\{\epsilon_t \epsilon_t^T\} = R \in \mathbb{R}^{n\times n}$$
$$z_t = C_t x_t + \delta_t \quad \text{with} \quad \mathbb{E}\{\delta_t \delta_t^T\} = Q \in \mathbb{R}^{k\times k}$$

Where system state is $x_t \in \mathbb{R}^n$, state update matrix is $A_t \in \mathbb{R}^{n\times n}$. $u_t \in \mathbb{R}^m$ is the control, with $B_t \in \mathbb{R}^{n\times m}$, and process noise $\epsilon_t \in \mathbb{R}^n$. Measurement is $z_t \in \mathbb{R}^k$ with $C_t \in \mathbb{R}^{k\times n}$, and measurement noise $\delta_t \in \mathbb{R}^k$. $k \leq n$. The subscript t indicates that the quantity depends on t (time varying).

1. Write a MATLAB script to simulate a Kalman tracking of a system with. 10 points
   (a) $1 \leq t \leq T$, with $T = 240$.
   (b) $A = \begin{pmatrix} 0.8000 & 0.0180 & -0.0146 \\ -0.0180 & 0.8000 & -0.0774 \\ 0.0146 & 0.0774 & 0.8000 \end{pmatrix}$
   (c) $B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
   (d) $R = \begin{pmatrix} 0.1 & 0.02 & 0.01 \\ 0.02 & 0.1 & -0.02 \\ 0.01 & -0.02 & 0.1 \end{pmatrix}$
   (e) $Q = \begin{pmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{pmatrix}$
   (f) Arbitrary initial state x_0 drawn from $\mathcal{N}(0, I_{n\times n})$
   (g) $u_t = \begin{pmatrix} \cos\left(\frac{6\pi t}{T}\right) \\ \sin\left(\frac{12\pi t}{T}\right) \\ 4\left(1 - \frac{\sin\left(\frac{2\pi t}{T}\right)}{\left|\sin\left(\frac{2\pi t}{T}\right)\right|}\right) \end{pmatrix}$
   (h) $z_t = \begin{pmatrix} x_{1,t} \\ -0.1x_{1,t} + 0.9x_{2,t} \end{pmatrix}$

2. Repeat part 1 (above) with $B_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\mu_0 = 0$ and $x_0 = 0$. Comment on the result. 10 points

3. Repeat part 1 (above) with both noises ϵ_t and δ_t from uniform distribution with zero mean and covariances $R = 0.01 I_{n\times n}$ and $Q = 0.01 I_{k\times k}$. Comment on the result. 10 points