Fall 2020 Quiz 12 Problems MTH 3720

You must show **all** work to receive full credit. No . All work is to be your own. Due: 11/16/2020
Be neat & organized, & use correct notation. This will be a closed books & notes quiz. 10:00 - 10:50

1. Real and Distinct Roots Answer questions (a)-(e) for each of the given *initial value problems*

$$y'' + 2y' - 3y = 0, \quad y(0) = 2, \quad y'(0) = 3 \tag{1}$$
$$y'' + 3y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = 3 \tag{2}$$
$$2y'' - 3y' + y = 0, \quad y(0) = 2, \quad y'(0) = 3 \tag{3}$$
$$y'' + 7y' + 12y = 0, \quad y(0) = 2, \quad y'(0) = 3 \tag{4}$$

(a) Find the general solution of the ODE above. 3 points

(b) Solve the *initial value problem* above. 3 points

(c) Reformulate the *initial value problem* above as a *dynamical system* $\boxed{\mathbf{x}' = \mathbf{A}\mathbf{x}, \quad \mathbf{x}(0) = \mathbf{x}_0}$ 4 points

(d) Rewrite the solution to the equation in (a) as a solution to the system in part (c), that is as $\boxed{\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)}$ 5 points

(e) Hand draw the phase portrait of the system in (c) using the results obtained in (d). 5 pts

(f) Use MATLAB's pplane8.m[1] to draw the phase portrait of the system. Use $-4 \le x_1 \le 4$ and $-4 \le x_2 \le 4$. Generate enough trajectories to display the type of phase portrait clearly. Display all of the straight (unbent) trajectories along the eigenvectors. Your result should look good enough to be accepted for a publication in a scientific journal. There is no partial credit for this part, so make sure it shines. 10 points

(g) Use the discrete time analog $\mathbf{x}_{i+1} = \mathbf{F}(\mathbf{x}_i)$ of the dynamical system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ to plot discrete time trajectories in MATLAB. Your result should resemble the graph obtained using pplane8.m, otherwise there is no partial credit for this part. Use axis([-4 4 -4 4]) if necessary. 20 points

2. Complex Conjugate Roots Answer questions (a)-(e) for each of the given *initial value problems*

$$y'' - 2y' + 2y = 0, \quad y(0) = 3, \quad y'(0) = 2 \tag{5}$$
$$y'' - 2y' + 6y = 0, \quad y(0) = 3, \quad y'(0) = 2 \tag{6}$$
$$y'' + 2y' + 2y = 0, \quad y(0) = 3, \quad y'(0) = 2 \tag{7}$$
$$4y'' + 9y = 0, \quad y(0) = 3, \quad y'(0) = 2 \tag{8}$$

(a) Find the general solution of the ODE above. 3 points

(b) Solve the *initial value problem* above. 3 points

(c) Reformulate the *initial value problem* above as a *dynamical system* $\boxed{\mathbf{x}' = \mathbf{A}\mathbf{x}, \quad \mathbf{x}(0) = \mathbf{x}_0}$ 4 points

(d) Rewrite the solution to the equation in (a) as a solution to the system in part (c), that is as $\boxed{\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)}$ 5 points

(e) Hand draw the phase portrait of the system in (c) using the results obtained in (d). 5 pts

[1] http://shawasnm.faculty.udmercy.edu/ODEs/pplane8/pplane8.m

(f) Use MATLAB's pplane8.m[2] to draw the phase portrait of the system. Use $-4 \leq x_1 \leq 4$ and $-4 \leq x_2 \leq 4$. Generate enough trajectories to display the type of phase portrait clearly. Your result should look good enough to be accepted for a publication in a scientific journal. There is no partial credit for this part, so make sure it shines. 10 points

(g) Use the discrete time analog $\mathbf{x}_{i+1} = \mathbf{F}(\mathbf{x}_i)$ of the dynamical system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ to plot discrete time trajectories in MATLAB. Your result should resemble the graph obtained using pplane8.m, otherwise there is no partial credit for this part. Use axis([-4 4 -4 4]) if necessary. 20 points

3. Repeated Roots Answer questions (a)-(e) for each of the given *initial value problems*

$$y'' - 2y' + y = 0, \;\; y(0) = 5, \;\; y'(0) = 3 \tag{9}$$
$$9y'' + 6y' + y = 0, \;\; y(0) = 3, \;\; y'(0) = 5 \tag{10}$$
$$4y'' - 4y' + y = 0, \;\; y(0) = 5, \;\; y'(0) = 3 \tag{11}$$
$$4y'' + 12y' + 9y = 0, \;\; y(0) = 3, \;\; y'(0) = 5 \tag{12}$$

(a) Find the general solution of the ODE above. 3 points

(b) Solve the *initial value problem* above. 3 points

(c) Reformulate the *initial value problem* above as a *dynamical system* $\boxed{\mathbf{x}' = \mathbf{A}\mathbf{x}, \quad \mathbf{x}(0) = \mathbf{x}_0}$ 4 points

(d) Rewrite the solution to the equation in (a) as a solution to the system in part (c), that is as $\boxed{\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)}$ 5 points

(e) Show that the ODE solution obtained in (d) can satisfy arbitrary initial conditions at any time t. 5 points

(f) Use MATLAB's pplane8.m[3] to draw the phase portrait of the system. Use $-4 \leq x_1 \leq 4$ and $-4 \leq x_2 \leq 4$. Generate enough trajectories to display the type of phase portrait clearly. Your result should look good enough to be accepted for a publication in a scientific journal. There is no partial credit for this part, so make sure it shines. 10 points

(g) Use the discrete time analog $\mathbf{x}_{i+1} = \mathbf{F}(\mathbf{x}_i)$ of the dynamical system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ to plot discrete time trajectories in MATLAB. Your result should resemble the graph obtained using pplane8.m, otherwise there is no partial credit for this part. Use axis([-4 4 -4 4]) if necessary. 20 points

[2] http://shawasnm.faculty.udmercy.edu/ODEs/pplane8/pplane8.m
[3] http://shawasnm.faculty.udmercy.edu/ODEs/pplane8/pplane8.m