Fall 2020 Quiz 13 Problems MTH 372

You must show **all** work to receive full credit. No 🔊. All work is to be your own. Date: 11/20/2020
Be neat & organized, & use correct notation. This will be a closed books & notes quiz. 10:00 - 10:50

§4.4 Mechanical and Electrical Vibrations. Problems from pages 262–264

1. Assume the system described by the equation $my'' + \gamma y' + ky = 0$ is either critically damped or overdamped. Show that the mass can pass through the equilibrium position at most once, regardless of the initial conditions. 20 points

2. Assume that the system described by the equation $my'' + \gamma y' + ky = 0$ is critically damped and that the initial conditions are $y(0) = y_0$ and $y'(0) = v_0$. If $v_0 = 0$, show that $y \to 0$ as $t \to \infty$ but that y is never zero. If y_0 is positive, determine a condition on v_0 that will ensure that the mass passes through its equilibrium position after it is released. 20 points

3. A series circuit has a capacitor of 0.25×10^{-6} farad and an inductor of 1 henry. If the initial charge on the capacitor is 10^{-6} coulomb and there is no initial current, find the charge on the capacitor at any time t. 20 points

4. A mass weighing 8 lb stretches a spring 1.5 in. The mass is also attached to a damper with coefficient γ. Determine the value of γ for which the system is critically damped. Be sure to give the units for γ. 20 points

5. Show that the period of motion of an undamped vibration of a mass hanging from a vertical spring is $2\pi\sqrt{\frac{L}{g}}$, where L is the elongation of the spring due to the mass and g is the acceleration due to gravity. 20 points

6. A certain vibrating system satisfies the equation $my'' + \gamma y' + ky = 0$. Find the value of the damping coefficient γ (depending on m and k) for which the quasi period of the damped motion is 60% greater then the period of the corresponding undamped motion. 20 points

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§4.5 Nonhomogeneous Equations; Method of Undetermined Coefficients. Problems from page 273

1. Use the method of *undetermined coefficients* to find the general solution of the given ODE. 20 points

$$y'' + 2y' + y = 2e^{-t}$$

2. Use the method of *undetermined coefficients* to find the general solution of the given ODE. 20 points

$$u'' + \omega_0^2 u = \cos \omega t, \qquad \omega^2 \neq \omega_0^2$$

3. Use the method of *undetermined coefficients* to find the general solution of the given ODE. 20 points

$$u'' + \omega_0^2 u = \cos \omega_0 t$$

4. Use the method of *undetermined coefficients* to find the solution of the IVP 20 points

$$y'' + y' - 2y = 2t, \qquad y(0) = 0, \;\; y'(0) = 1$$

5. Use the method of *undetermined coefficients* to find the solution of the IVP 20 points

$$y'' + 4y = 3 \sin 2t, \qquad y(0) = 2, \;\; y'(0) = -1$$

6. Use the method of *undetermined coefficients* to find the solution of the IVP 20 points

$$y'' + 4y = t^2 + 3e^t, \qquad y(0) = 0, \;\; y'(0) = 2$$